

# A Mercury orientation model including non-zero obliquity and librations

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**Abstract** Planetary orientation models describe the orientation of the spin axis and prime meridian of planets in inertial space as a function of time. The models are required for the planning and execution of Earth-based or space-based observational work, e.g. to compute viewing geometries and to tie observations to planetary coordinate systems. The current orientation model for Mercury is inadequate because it uses an obsolete spin orientation, neglects oscillations in the spin rate called longitude librations, and relies on a prime meridian that no longer reflects its intended dynamical significance. These effects result in positional errors on the surface of  $\sim 1.5$  km in latitude and up to several km in longitude, about two orders of magnitude larger than the finest image resolution currently attainable. Here we present an updated orientation model which incorporates modern values of the spin orientation, includes a formulation for longitude librations, and restores the dynamical significance to the prime meridian. We also use modern values of the orbit normal, spin axis orientation, and precession rates to quantify an important relationship between the obliquity and moment of inertia differences.

**Keywords** Planets rotation · Mercury · Gravity

## 1 Introduction

The IAU Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites (WGCCRE) has published orientation models for Mercury since 1980. The availability of new Earth-based and spacecraft data warrants a revision to the existing model. Our intent is to summarize recent advances and to propose an updated model for consideration by the WGCCRE. We examine three limitations to the current model: (1) The

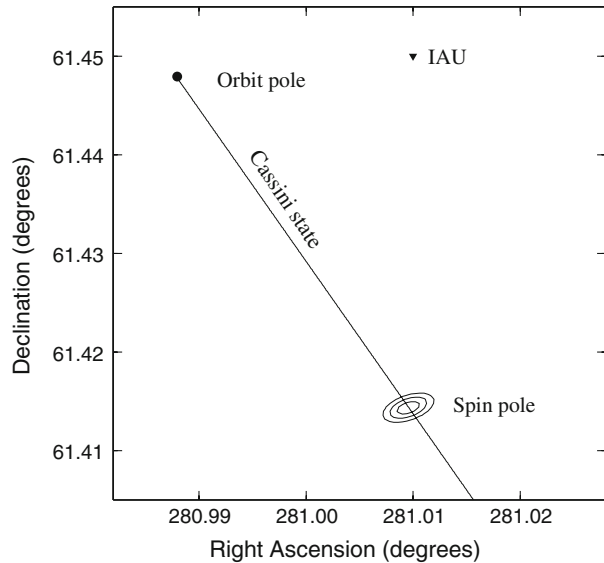
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This paper is dedicated to the memory of Steven J. Ostro, vibrant scientist and dear friend.

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**Fig. 1** Mercury spin and orbit pole orientations at epoch J2000 in J2000 equatorial coordinates. The IAU value for the spin axis orientation (*filled triangle*) is reportedly chosen to be perpendicular to the orbital plane, but does not coincide with modern values of the orbit pole (*filled circle*). The IAU value differs from the measured spin pole orientation (*contours*) by  $\sim 0.04^\circ$ , an unacceptably large offset for precision work. Adapted from Margot et al. (2007)



IAU spin orientation (Seidelmann et al. 2007) is based on assumptions made in 1980 and does not reflect current knowledge (Fig. 1); (2) The model does not incorporate longitude librations which have been shown recently to be measurable (Margot et al. 2007); (3) Updates in the 1994 and 2000 reports (Table 1) have shifted the prime meridian  $\sim 0.2^\circ$  away from the dynamical location intended in the early reports.

## 2 Spin and orbit orientations

The current IAU values for the spin axis orientation (Table 1) can be traced directly to the values chosen in 1980, when perpendicularity to the orbital plane was assumed. All subsequent reports list the original values essentially unchanged.

A modern value for the orientation of the orbit pole can be derived from published Keplerian elements (Standish undated). The elements are valid for the time interval 1800 AD–2050 AD and yield the value ( $\alpha = 280.9879^\circ$ ,  $\delta = 61.4478^\circ$ ) at epoch J2000. As an independent check we computed the evolution of the orbit pole using DE408 data over a  $\pm 100$  year period centered on J2000. We obtained a nearly identical orbit pole ( $\alpha = 280.9880^\circ$ ,  $\delta = 61.4478^\circ$ ) and precession values ( $\dot{\alpha} = -0.0328^\circ/\text{cy}$ ,  $\dot{\delta} = -0.0049^\circ/\text{cy}$ ) that confirm the IAU rates.

A modern value for the orientation of the spin axis ( $\alpha = 281.0097^\circ$ ,  $\delta = 61.4143^\circ$ ) was measured with radar by Margot et al. (2007) on the basis of a technique proposed by Holin (1988, 1992). Twenty-one measurements obtained from 2002 to 2006 at a wide range of geometries yield a robust obliquity value of  $2.11 \pm 0.1$  arcminutes, precisely within the range of theoretical expectations (Peale 1988; Peale et al. 2002). Although data analysis does not assume the Cassini state in any way, the spin axis uncertainty contours fall on the locus of possible Cassini state positions defined by the orbit pole and the Laplace pole of Yseboodt and Margot (2006) ( $\alpha_L = 273.7239^\circ$ ,  $\delta_L = 69.5263^\circ$ ). If one assumes the Cassini state the spin axis rates can be set to the orbit precession rates to a very good approximation.

The  $\sim 300,000$  year precession of the orbit and spin orientations about the Laplace pole is noticeable. The predicted spin axis orientation at the time of MESSENGER orbit insertion

**Table 1** Mercury orientation models as published in WGCCRE reports

Year	Reference	Orientation $\alpha_0, \delta_0, W[^\circ]$	Notes
1980	<a href="#">Davies et al. (1980)</a>	280.9 – 0.033 T 61.4 – 0.005 T 184.74 + 6.1385025 d	Equinox B1950, epoch J1950 <sup>a</sup>
1982	<a href="#">Davies et al. (1983)</a>	281.02 – 0.033 T 61.45 – 0.005 T 329.71 + 6.1385025 d	Equinox J2000, epoch J2000 <sup>b</sup>
1985	<a href="#">Davies et al. (1986)</a>	281.01 – 0.003 T 61.45 – 0.005 T 329.71 + 6.1385025 d	Typo in RA rate <sup>c</sup>
1988	<a href="#">Davies et al. (1989)</a>		No change
1991	<a href="#">Davies et al. (1992)</a>		No change
1994	<a href="#">Davies et al. (1996)</a>	281.01 – 0.003 T 61.45 – 0.005 T 329.68 + 6.1385025 d	Typo propagated  d
1997			No report
2000	<a href="#">Seidelmann et al. (2002)</a>	281.01 – 0.033 T 61.45 – 0.005 T 329.548 + 6.1385025 d	Typo fixed  e
2003	<a href="#">Seidelmann et al. (2005)</a>		No change
2006	<a href="#">Seidelmann et al. (2007)</a>		No change

The right ascension and declination ( $\alpha, \delta$ ) define the spin axis (see Fig. 1) while W gives the rotational phase. The prime meridian is defined such that the crater Hun Kal lies on the  $20^\circ$  meridian. Here T is the interval in Julian centuries (of 36525 days) from the standard epoch, and d is the interval in days (of 86400 SI seconds) from the standard epoch, with epochs defined in Barycentric Dynamical Time (TDB)

<sup>a</sup> Original values assume perpendicularity to orbital plane as it was known in 1980

<sup>b</sup> If one precesses the 1980 spin axis from the 1950.0 epoch to the 2000.0 epoch with the given rates, then converts to J2000 equatorial coordinates, one finds the values listed in the 1982 report

<sup>c</sup> There is no explanation given for the change in the last digit of  $\alpha_0$  in the 1985 report

<sup>d</sup> “The new value for the W0 of Mercury was the result of a new control network computation by Davies and Colvin (RAND) that included the determination of the focal lengths of the Mariner 10 cameras”

<sup>e</sup> “The new value for the W0 of Mercury was the result of a new control network computation by [Robinson et al. \(1999\)](#)”

on 18 March 2011 is ( $\alpha_{\text{MOI}} = 281.0061^\circ, \delta_{\text{MOI}} = 61.4136^\circ$ ), about 7 arcseconds away from the J2000 epoch position.

### 3 Librations in longitude

For high precision work the orientation of the planet must include the forced librations in longitude with a period of  $\sim 88$  days ([Peale 1988](#)) and current best-fit amplitude of  $\sim 36$  arcseconds ([Margot et al. 2007](#)). Failure to account for this motion can result in positional inaccuracies of  $\sim 0.01^\circ$  in longitude, or  $\sim 425$  m at the equator.

With the assumption that the spin axis is perpendicular to the orbital plane, the longitudinal orientation of a permanently deformed body orbiting in the gravitational potential of a central body is governed by a tidal torque equation (e.g. [Goldreich and Peale 1966](#); [Wisdom](#)

**Table 2** Coefficients in the series solution to the libration angle  $\gamma$  and their numerical values

$k$	$f_k(e)$	Value
1	$1 - 11e^2 + \frac{959e^4}{48} - \frac{3641e^6}{288}$	+0.569638
2	$-\frac{e}{8} - \frac{421e^3}{96} + \frac{32515e^5}{3072}$	-0.0599438
3	$-\frac{533e^4}{144} + \frac{4609e^6}{480}$	-0.0058920
4	$\frac{e^3}{768} - \frac{57073e^5}{15360}$	-0.0013548
5	$\frac{e^4}{600} - \frac{18337e^6}{4500}$	-0.0003051

et al. 1984; Murray and Dermott 1999)

$$\ddot{\theta} + \frac{3}{2} \left( \frac{B - A}{C} \right) \frac{GM_{\odot}}{r^3} \sin 2(\theta - f) = 0, \tag{1}$$

where  $\theta$  is the angular position of the long axis and  $f$  is the true anomaly, both measured with respect to the same inertial line,  $A < B < C$  are the moments of inertia,  $G$  is the gravitational constant,  $M_{\odot}$  the mass of the central body, and  $r$  the distance between the two centers of mass. The equation is not tractable analytically but for bodies in a spin-orbit resonance we can provide a very good approximation to  $\theta$  with the sum of a linear function of time (capturing the resonant spin) and a trigonometric series (capturing small deviations with respect to the resonant spin).

For Mercury, the mean planetary spin rate  $\langle \dot{\theta} \rangle$  is  $3/2$  the mean motion  $n$ , and it is customary to define a small libration angle  $\gamma$  such that

$$\gamma = \theta - \frac{3}{2}M, \tag{2}$$

$$\dot{\gamma} = \dot{\theta} - \frac{3}{2}n, \tag{3}$$

where  $M = n(t - t_0)$  is the mean anomaly and  $t_0$  is the epoch of pericenter passage. The libration equation can be rewritten

$$\ddot{\gamma} + \frac{3}{2}n^2 \left( \frac{B - A}{C} \right) \left( \frac{a}{r} \right)^3 \sin (2\gamma + 3M - 2f) = 0. \tag{4}$$

To obtain an approximate solution we first expand the sine factor in the small angle  $\gamma$  and retain only the dominant term. We then expand the non-linear function of time  $\left(\frac{a}{r}\right)^3 \sin (3M - 2f)$  as a trigonometric series of the mean anomaly using standard techniques (Murray and Dermott 1999). Finally we integrate twice with respect to time and find a solution of the form

$$\gamma \cong \frac{3}{2} \left( \frac{B - A}{C} \right) \sum_k f_k(e) \sin(kM), \tag{5}$$

where the first few  $f_k(e)$  are functions of the orbital eccentricity only (Table 2).

Comparison of the amplitude of the truncated ( $k \leq 5$ ) series solution with direct numerical integrations show that the solution is valid everywhere to 0.3%.

The orientation of Mercury is found by combining Eqs. 2 and 5:

$$\theta = \frac{3}{2}n(t - t_0) + \frac{3}{2} \left( \frac{B - A}{C} \right) \sum_k f_k(e) \sin(kn(t - t_0)) \tag{6}$$

where we have made the time dependence explicit with the substitution  $M = n(t - t_0)$ .

The orientation  $\theta$  is measured in the plane of the orbit with respect to the Sun-Mercury line at perihelion. The IAU defines the prime meridian by an angle  $W_0$  measured easterly along the body's equator from the intersection of the body's equator and International Celestial Reference Frame (ICRF) equator. To relate  $\theta$  and  $W_0$ , we solved angles in the spherical triangle defined by the equinoctial point, ascending node of Mercury's orbit, and intersection of Mercury's equator and ICRF equator. With values of the orbital parameters suitable at epoch J2000 (Standish undated), this yields  $W_0 = 329.75^\circ$ .

#### 4 Prime meridian

Because Mercury is in a spin-orbit resonance in which it spins on its axis three times for every two revolutions around the Sun, the planet always presents one of two longitudes to the Sun at perihelion. These longitudes correspond to the axis of minimum moment of inertia because tidal torques have the effect of aligning the "long" axis of the planet with the direction of the Sun at perihelion. This provides a very natural choice for the prime meridian.

Early WGCCRE reports clearly intended to define the prime meridian with the dynamical significance in mind, as evidenced by the value of  $W_0 = 329.71^\circ$  (Table 1), which matches the sub-solar point at perihelion to  $\sim 0.04^\circ$ . After new network computations, the value of  $W_0$  was lowered to  $329.68^\circ$  and  $329.55^\circ$  in the 1994 and 2000 reports, respectively, presumably to maintain crater Hun Kal on the  $20^\circ$  meridian (Hun Kal means twenty in the Mayan mathematical system). The unfortunate consequence of these updates is that the current IAU prime meridian has lost its dynamical significance and is now  $\sim 0.2^\circ$  ( $\sim 8$  km in longitude) away from the long axis. This is more than a geographical inconvenience. Non-diagonal elements of the inertia tensor and corresponding coefficients in spherical harmonic expansions to the gravity field will be zero if the coordinate system is aligned with the principal axes, but not otherwise. Should the WGCCRE wish to preserve the intent of the early reports and restore the dynamical significance to the prime meridian, then a value of  $W_0$  closer to  $329.75^\circ$  would be more appropriate. This could easily be accomplished by slightly modifying the longitude of the current defining feature Hun Kal, or by selecting a suitable feature from new high resolution imagery to define the prime meridian near zero longitude.

Although Earth's prime meridian was chosen among more than ten possibilities at the 1884 International Meridian Conference, the WGCCRE may well take the position that the prime meridian, once chosen, should be immutable. This choice would protect against further adjustments to the prime meridian and against similar adjustments on other bodies. In that case, serious consideration should be given to providing a transformation matrix between the geographically defined and the dynamically defined systems.

#### 5 Recommended model

We used the current best estimate of  $\left(\frac{B-A}{C}\right) = 2.03 \times 10^{-4}$  (Margot et al. 2007) and the values in Table 2 to arrive at the model in Table 3. The small changes to  $\alpha_0$ ,  $\delta_0$  compared to the IAU 2006 model do not affect  $W_0$  at its current level of precision. We chose the  $W_0$  value that restores the dynamical significance to the prime meridian. A different  $W_0$  value can be used, in which case the geographically defined system would not coincide with the frame defined by dynamics.

Long-period librations are not included in the model because such librations are unconfirmed. It will take observations over most of their  $\sim 12$  year period to establish their presence

**Table 3** Recommended model for the orientation of Mercury

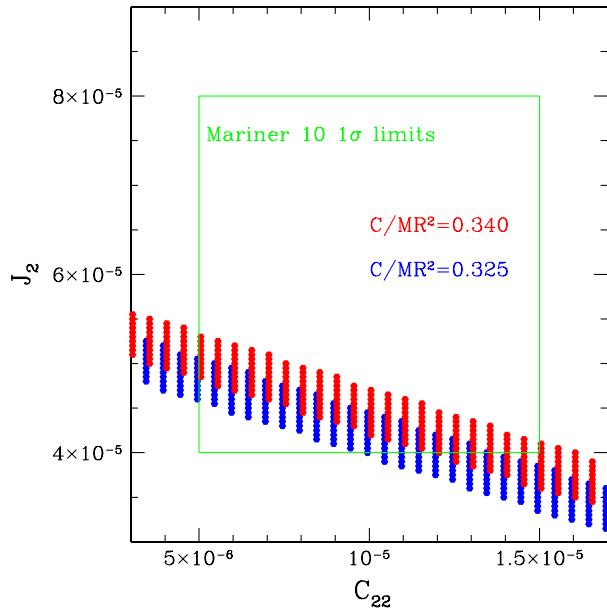
$$\begin{aligned} \alpha &= 281.0097 - 0.0328 T \\ \delta &= 61.4143 - 0.0049 T \\ W &= 329.75 + 6.1385025 d \\ &\quad + 0.00993822 \sin(M1) \\ &\quad - 0.00104581 \sin(M2) \\ &\quad - 0.00010280 \sin(M3) \\ &\quad - 0.00002364 \sin(M4) \\ &\quad - 0.00000532 \sin(M5) \end{aligned}$$

where

$$\begin{aligned} M1 &= 174.791086 + 4.092335 d \\ M2 &= 349.582171 + 8.184670 d \\ M3 &= 164.373257 + 12.277005 d \\ M4 &= 339.164343 + 16.369340 d \\ M5 &= 153.955429 + 20.461675 d \end{aligned}$$

Angles are expressed in degrees, and T and d are defined as in Table 1

**Fig. 2** Values of  $J_2 = (C - A)/MR^2$  and  $C_{22} = (B - A)/4MR^2$  allowed by the  $(2.11 \pm 0.1)^\circ$  obliquity for two assumed values of the polar moment of inertia (*red* and *blue*). Values derived from Mariner 10 radio science data (Anderson et al. 1987) are shown in *green* (color figure online)



and quantify their amplitude and phase. The long-period librations should damp on  $10^5$  year timescales (Peale 2005) unless they are excited by a internal mechanism or by a fortuitous value of  $(\frac{B-A}{C})$  that allows for resonant forcing by Jupiter (Peale et al. 2007; Dufey et al. 2008; Peale et al. 2009; Yseboodt et al. 2009). The addition of long-term librations would complicate the model as the angles in the additional terms would depend on the value of  $(\frac{B-A}{C})$ . In the proposed model only the coefficients in the trigonometric series depend linearly on  $(\frac{B-A}{C})$ , so it is straightforward to incorporate improved estimates of the moment differences.

## 6 Geophysical significance

The values of the orbit orientation, spin axis orientation, and precession rates described in this paper allow us to quantify an important relationship between the obliquity and moment of inertia differences. This relationship exists for planetary bodies in a Cassini state (Peale 1988). For reasonable assumptions of the polar moment of inertia, we illustrate the finite set of gravitational harmonic coefficients that are allowed by the occupancy of the Cassini state and by the observed obliquity (Fig. 2).

## 7 Conclusions

We propose a new orientation model for Mercury. The model uses modern values for the spin orientation and precession rates, incorporates longitude librations, and restores the dynamical significance to the prime meridian.

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